Null-Boundary Correspondence and Non-Synchronous Cosmology in a Layered Geometric Framework

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We develop a structured extension of general relativity in which an evolving three-dimensional spatial ontology is represented through distinct Lorentzian spacetime projections. Building on recent results demonstrating the null–temporal degeneracy of event horizons, we formulate Cosmological Relativity (CR) as a layered geometric framework in which spatial evolution is primary and spacetime geometry is representational. Within this framework, we establish two main results.

First, we prove a *Null–Boundary Correspondence Theorem*: any null hypersurface arising as the limit of infalling timelike worldlines admits a diffeomorphic correspondence to the initial slice of a Schwarzschild–de Sitter (SdS) cosmological projection under a CR causal reassignment that preserves the underlying foliation. This mapping clarifies how distinct Lorentzian metrics defined on the same smooth manifold may represent the same ontological layer through different causal assignments.

Second, we construct a non-synchronous cosmological model obtained by causal reinterpretation of de Sitter space. This SdS representation preserves the cosmic foliation while reassigning null and timelike congruences in a manner compatible with its Lorentzian structure, yielding an expanding 3-sphere whose radius evolves with the exact $\sinh^{2/3}$ law of flat ΛCDM. The resulting observational expansion history agrees with standard cosmology despite the absence of global synchrony and without invoking dynamical evolution governed by matter density.

Together, these results demonstrate how null boundaries and non-synchronous cosmological expansions arise naturally as representational features of a layered geometric ontology. The framework preserves full diffeomorphism invariance and the Einstein field equations, clarifies the structural freedom underlying relativistic projections, and aligns the SdS construction with symmetry correspondences in the relevant Lorentz and de Sitter groups. This situates the observed large-scale cosmological expansion within the internal geometric logic of general relativity itself.

THE LAYERED GEOMETRIC FRAMEWORK OF COSMOLOGICAL RELATIVITY

Cosmological Relativity (CR) is introduced as a formal augmentation of general relativity. It does not modify Einstein's field equations, the Lorentzian metric, or the causal structure of spacetime. Instead, it introduces additional geometric structure to distinguish between the representation of events and the ontological structure of spatial existence.

Underlying Spacetime Structure

Axiom 1 (Lorentzian Spacetime)**.** *Let (M, g) be a smooth, time-orientable Lorentzian manifold. The metric g defines the standard causal structure, including timelike, null, and spacelike curves, null cones, and all standard constructions of general relativity [1, 2]. This structure is retained without modification.*

An *event* is defined as a point $p \in M$. The manifold M represents the totality of events that occur.

Occurrence and Representation

Definition 1 (Occurrence)**.** An event $p \in M$ is said to *occur* relative to a temporal slicing if p lies on, or is a limit point of, a spacelike hypersurface belonging to that slicing.

This notion of occurrence is purely representational and does not by itself assign ontological status to events.

Layered Geometric Framework

Axiom 2 (Layered Geometric Framework)**.** *There exists a one-parameter family of smooth three-dimensional manifolds*

$$\mathcal{U} = \{\mathcal{S}_t \mid t \in \mathbb{R}\},$$

called ontological spatial layers*, each equipped with a Riemannian metric h_t. The parameter t is a global ordering parameter referred to as* cosmic time*. It is not defined operationally and is not identified with any coordinate time on M.*

Remark 1. The introduction of a global cosmic-time parameter in Cosmological Relativity represents an augmentation of the representational structure of general relativity. This parameter is not assumed to be orthogonal to the spatial geometry, nor does it coincide with any coordinate time in a preconceived spacetime foliation. As shown in the accompanying work, the null–temporal degeneracy of event horizons identifies a canonical limiting temporal orientation within general relativity itself [**?**], suggesting that a non-orthogonal global time parameter is already structurally motivated by the collapse geometry. Here, however, this structure is introduced as a hypothesis of CR; its justification emerges later through the SdS construction and null–boundary correspondence. A further clarification of the representational distinction between the manifold, its Lorentzian metric, and the ontological layering is given in the fixed manifold representational freedom and metric reassignment section below, where the role of metric reassignment in encoding causal structure is made explicit.

Representability Condition

Axiom 3 (Diffeomorphic Representability)**.** *For each $t \in \mathbb{R}$, there exists a smooth spacelike hypersurface $\Sigma_t \subset M$ and a diffeomorphism*

$$\Phi_t : \mathcal{S}_t \to \Sigma_t.$$

No uniqueness or physical preference of Σ_t is assumed.

Axiom 4 (Non-Identity of Ontology and Representation)**.** *The existence of the diffeomorphism Φ_t does not imply ontological equivalence between $\mathcal{S}_t$ and Σ_t. The spacelike hypersurface Σ_t represents the ontological layer $\mathcal{S}_t$ but does not constitute it.*

Ontological Simultaneity

Definition 2 (Ontological Simultaneity)**.** Two events $p, q \in M$ are said to be ontologically simultaneous if there exists a $t \in \mathbb{R}$ such that

$$p, q \in \Phi_t(\mathcal{S}_t).$$

Ontological simultaneity is not identified with Einstein synchrony, has no operational definition, and introduces no new causal relations.

Compatibility with General Relativity

Remark 2. All physical observables, causal relations, and empirical predictions remain functions solely of the Lorentzian structure (M, g). The layered geometric framework introduces no additional causal structure and preserves full diffeomorphism invariance.

CONSISTENCY OF LAYERED ONTOLOGY AND SPACETIME PROJECTIONS

We now establish that the axioms of Cosmological Relativity are internally consistent and compatible with standard solutions of general relativity.

Event Horizons

Definition 3 (Event Horizon)**.** The event horizon $\mathcal{H}^+$ is defined exactly as in general relativity by

$$\mathcal{H}^+ = \partial J^-(\mathcal{I}^+)$$

[3].

No modification of this definition is made in CR.

Imported Result from General Relativity

The following result is established in the accompanying work on the null–temporal degeneracy of event horizons [4].

Theorem 1 (Asymptotic Non-Intersection of Event Horizons)**.** *For any smooth temporal function adapted to an exterior observer, no finite value of the temporal parameter intersects the event horizon $\mathcal{H}^+$. All such slices approach $\mathcal{H}^+$ only asymptotically in the infinite-time limit.*

Smoothness of Ontological Layers

Theorem 2 (Smoothness of Ontological Spatial Layers)**.** *For every finite cosmic time t, the ontological spatial layer $\mathcal{S}_t$ is a smooth Riemannian manifold and contains no point diffeomorphic to an event-horizon generator or to a curvature singularity.*

Proof. By the representability axiom, $\mathcal{S}_t$ is diffeomorphic to a spacelike hypersurface $\Sigma_t \subset M$. By the asymptotic non-intersection theorem, no spacelike hypersurface corresponding to a finite temporal parameter intersects $\mathcal{H}^+$ or any curvature singularity. Therefore, Σ_t lies entirely in the regular region of spacetime, and so $\mathcal{S}_t$ is smooth. □

Consistency Statement

Corollary 1. *Cosmological Relativity admits smooth ontological spatial layers for all finite cosmic times and is therefore internally consistent with the causal and geometric structure of general relativity.*

Remark 3. This result does not assert the nonexistence of singularities in spacetime. It asserts only that such structures do not occur on any finite ontological spatial layer and therefore do not puncture the ontological structure of the universe at finite cosmic time.

APPLICATIONS OF THE LAYERED GEOMETRIC FRAMEWORK

Cosmological Relativity permits multiple spacetime representations of a single underlying evolving spatial geometry. In this section we formalize how standard relativistic spacetimes arise as distinct projections of the same layered geometric framework.

Projection Principle

Axiom 5 (Projection Principle)**.** *Let $(\mathcal{S}_t, h_{ij}(t))$ be an ontological spatial layer in the layered geometric framework. A spacetime (M, g) is said to be a valid projection of $(\mathcal{S}_t, h_{ij}(t))$ if there exists a foliation $\{\Sigma_t\}$ of M and a family of diffeomorphisms $\Phi_t : \mathcal{S}_t \to \Sigma_t$ such that the causal structure induced by g faithfully encodes signal propagation on $(\mathcal{S}_t, h_{ij}(t))$.*

No uniqueness of the projection is assumed.

Representational Non-Uniqueness of Spacetime Geometry

Proposition 1. *Distinct Lorentzian metrics $g_{\mu\nu}$ may arise as valid spacetime projections of the same evolving spatial geometry $(\mathcal{S}_t, h_{ij}(t))$.*

Remark 4. In CR, spacetime curvature is a property of the projection, not of the underlying ontology. Ontological curvature is encoded exclusively in the intrinsic geometry of $h_{ij}(t)$. A detailed discussion of how distinct Lorentzian metrics on the same manifold may encode different causal assignments while preserving the underlying foliation is provided in the fixed manifold representational freedom and metric reassignment section below.

Flat Spacetime Projections

Proposition 2 (Minkowski Projection)**.** *If $(\mathcal{S}_t, h_{ij}(t))$ is diffeomorphic to $\mathbb{R}^3$ for all t, then there exists a projection of the layered geometry yielding Minkowski spacetime $(M, \eta_{\mu\nu})$, independent of the intrinsic curvature of $h_{ij}(t)$.*

Remark 5. This projection preserves causal propagation but does not require the spatial geometry to be Euclidean. Flat spacetime does not imply flat space in CR.

Curved Spacetime Projections

Proposition 3 (Schwarzschild Projection)**.** *If $(\mathcal{S}_t, h_{ij}(t))$ exhibits spherically symmetric curvature centered on a compact region, then there exists a projection of the layered geometry yielding the Schwarzschild spacetime [5] as an exact Lorentzian representation.*

Remark 6. Minkowski and Schwarzschild spacetimes are thus distinct causal projections of the same underlying evolving spatial geometry, not distinct ontological universes.

A fuller discussion of how distinct Lorentzian metrics on the same smooth manifold may encode different causal assignments while preserving the underlying ontological foliation is provided in the fixed manifold representational freedom and metric reassignment section below.

Gravitational Waves

Proposition 4. *In CR, gravitational waves correspond to propagating perturbations of the spatial metric $h_{ij}(t)$. Their representation as oscillations of the spacetime metric $g_{\mu\nu}$ is projection-dependent.*

Remark 7. This statement does not modify the dynamics of gravitational radiation. It reclassifies the ontological carrier of wave energy from spacetime geometry to evolving spatial geometry.

Time Travel

In standard interpretations of general relativity, the possibility of time travel arises from treating the spacetime manifold (M, g) as a fixed four-dimensional structure in which all events are equally real. In such a framework, coordinate-dependent simultaneity relations and global spacetime constructions may admit closed timelike curves [6] or apparent causal loops.

Cosmological Relativity excludes such structures not by imposing additional dynamical constraints, but by modifying the ontological interpretation of spacetime representation.

Proposition 5 (No Ontological Time Travel)**.** *In Cosmological Relativity, no physical process corresponds to backward temporal evolution or traversal of past ontological states.*

Proof. By construction, ontological states of the universe are represented by the layered geometric framework $\{\mathcal{S}_t\}$, which is totally ordered by the cosmic time parameter t. All physical evolution occurs as transitions between successive layers. Since no layer $\mathcal{S}_t$ exists prior to itself, and since spacetime (M, g) is treated as a representational projection of this structure rather than as ontologically fundamental, there exists no physical structure within which a trajectory could return to an earlier ontological layer. □

Corollary 2. *Closed timelike curves in the spacetime representation (M, g) have no ontological interpretation in CR and do not correspond to physically realizable processes.*

Remark 8. This result does not restrict the mathematical existence of closed timelike curves in certain spacetime solutions of Einstein's equations. It asserts only that such curves do not correspond to physical histories of the universe within the layered geometric framework.

Remark 9. Superluminal coordinate effects or frame-dependent simultaneity relations cannot generate backward temporal evolution in CR, since ontological simultaneity is defined independently of relativistic synchrony and does not admit causal inversion.

The Hole Argument and Event Identity

The hole argument highlights an apparent tension between diffeomorphism invariance and the individuation of events in general relativity. Since Einstein's equations are invariant under smooth diffeomorphisms, distinct mathematical models may assign different field values to the same manifold points without observational distinction [7].

In standard relationalist interpretations, this indeterminacy is resolved by denying ontological identity to spacetime points independently of the fields defined on them.

Cosmological Relativity resolves the hole argument by distinguishing event representation from event individuation.

Axiom 6 (Ontological Event Individuation)**.** *Events are individuated by their occurrence within ontological spatial* *layers $\mathcal{S}_t$, not by their coordinate representation in the spacetime manifold.*

Proposition 6 (Diffeomorphism-Invariant Identity)**.** *Diffeomorphic spacetime models that arise as projections of the same layered geometric framework represent the same ontological events.*

Proof. By the non-identity axiom of CR, the spacetime manifold serves only as a representational record of occurrences. Ontological identity is anchored in the layered structure $\{\mathcal{S}_t\}$, which remains invariant under diffeomorphic re-representations of spacetime. Therefore, diffeomorphic models correspond to the same ontological history. □

Corollary 3. *The indeterminacy highlighted by the hole argument reflects representational redundancy rather than ontological ambiguity.*

Remark 10. CR preserves full diffeomorphism invariance of the spacetime formalism while providing a fixed ontological basis for event identity. No privileged coordinate system or observable structure is introduced.

Remark 11. This resolution parallels the treatment of gauge freedom in other physical theories: representational redundancy does not imply indeterminacy of the physical system when an underlying invariant structure is specified.

The distinction between manifold structure, metric structure, and ontological layering will play a further role below in discussing how distinct Lorentzian metrics on the same manifold may represent different causal assignments while preserving the same underlying ontological content.

CR/FLRW as a Symmetric Projection

The Friedmann–Lemaître–Robertson–Walker (FLRW) cosmological model is conventionally derived in general relativity by imposing a set of strong symmetry assumptions: the existence of a global cosmic time, hypersurface orthogonality of the cosmic time flow, spatial isotropy, spatial homogeneity [8, 9], and dynamical evolution governed by the Einstein field equations. Together, these assumptions permit the construction of a highly symmetric Lorentzian metric whose scale factor satisfies the Friedmann equations.

In standard GR, this construction is often interpreted ontologically: the FLRW metric is taken to describe the real large-scale evolution of space itself. Cosmological Relativity rejects this identification. In CR, the FLRW metric is reinterpreted as a *projection* of an underlying evolving spatial geometry onto a highly symmetric foliation. The foliation records the causal appearance of expansion rather than defining the ontological structure of space.

As established in the layered geometric framework introduced above, CR distinguishes between ontological spatial layers $(\mathcal{S}_t, h_{ij}(t))$ and their spacetime representations. Real space may be locally curved, dynamically distorted, and inhomogeneous, while remaining diffeomorphic to the spatial slices of an FLRW foliation. The freedom to select among distinct Lorentzian metrics on the same underlying manifold, while holding the cosmic foliation fixed, is examined in more detail in the fixed manifold representational freedom and metric reassignment section below. This clarifies how synchronous and non-synchronous projections arise as representational choices rather than ontological distinctions.

The success of FLRW cosmology therefore does not require that space itself be homogeneous, isotropic, or synchronously expanding; it requires only that the causal projection of the evolving universe admits a maximally symmetric representation.

From this perspective, the assumption of hypersurface orthogonality of the cosmic time flow—central to standard FLRW cosmology—is revealed as a representational convenience rather than a physical necessity. CR allows for asynchronous evolution of real space while preserving observational isotropy for congruences propagating through the layered geometry. The familiar FLRW model is recovered as a limiting case corresponding to projections with maximal symmetry.

The empirical success of standard cosmology is thus preserved without elevating its symmetry assumptions to ontological principles. CR/FLRW retains all observational predictions of the standard model while clarifying that its symmetry content reflects properties of a particular projection, not of the underlying evolving universe.

Remark 12. The reliance of standard FLRW cosmology on synchronous, hypersurface-orthogonal cosmic time is a representational assumption rather than a physical necessity. Within CR, the cosmic foliation need not be orthogonal to the spatial layers, and the corollary established in the companion paper indicates that, under gravitational collapse, the canonical temporal direction determined by general relativity is generically non-orthogonal [4]. Thus the assumption of synchronous cosmic time is not only unnecessary within CR but is structurally disfavored by the horizon geometry of GR.

Remark 13. In standard cosmology, the FLRW framework is often treated as the minimal structure required to describe cosmic expansion. In Cosmological Relativity, FLRW instead emerges as a maximally symmetric limiting case within a broader class of admissible projections. By relaxing the requirement of hypersurface orthogonality while preserving cosmic time, isotropy, and causal structure, CR permits asynchronous but observationally isotropic cosmologies that remain fully consistent with relativistic causality and the Einstein field equations.

The consequences of this distinction become especially significant when considering singular events and null boundaries, such as those associated with gravitational collapse and event horizons. These structures motivate a deeper re-examination of the relationship between cosmological initial conditions and null causal geometry, which we take up in the non-synchronous SdS construction below.

Non-Synchronous SdS Cosmology via Causal Reassignment

The CR/FLRW framework decouples ontological evolution from spacetime representation while preserving the empirical successes of standard cosmology. However, CR/FLRW retains an inherited synchrony condition: spatial hypersurfaces of constant cosmic time expand uniformly across the entire universe. This synchronous evolution is not required by the Einstein field equations, nor is it implied by observation. In CR, synchrony is therefore a representational assumption rather than an ontological one.

We now construct a cosmological model in which cosmic expansion is not globally synchronous, yet the observational expansion history coincides exactly with that of a flat ΛCDM universe. The construction is based on a causal reinterpretation of de Sitter (dS) space consistent with the layered geometric framework introduced above. The geometric legitimacy of this reinterpretation, which involves selecting a distinct Lorentzian metric on the same underlying manifold while preserving the cosmic foliation, is discussed in the fixed manifold representational freedom and metric reassignment section below.

Causal Reassignment in de Sitter Space

Four-dimensional de Sitter space [10] may be represented as a one-sheeted hyperboloid embedded in five-dimensional Minkowski space, and it admits two independent rulings by null geodesics. In the CR framework, however, only one of these rulings plays a structural role: the family of null directions that matches the causal sense of the event-horizon generators arising during gravitational collapse. These null directions determine a unique bundle of future-directed null curves on the de Sitter hyperboloid, and this bundle is used to define the fundamental timelike congruence in the Schwarzschild–de Sitter (SdS) projection.

The complementary congruence used in the reassignment is *not* the second null ruling of the de Sitter hyperboloid. Instead, it is the congruence of worldlines that remain fixed on the expanding 3-spheres—curves of constant R in the embedding coordinates. These curves are spacelike in de Sitter space, but in the SdS projection they are reinterpreted as null geodesics and serve as the photon congruence. This exchange of causal roles is an instance of the representational freedom discussed below, where distinct Lorentzian metrics on the same manifold encode different causal assignments while preserving the underlying foliation. Thus the causal reassignment proceeds as follows:

- The future-directed null generators associated with the collapse-induced horizon structure are reinterpreted as the *timelike* worldlines of fundamental observers.

- The spacelike $R = \text{const.}$ curves, which trace points that remain at fixed locations on each 3-sphere, are reinterpreted as the *null* trajectories of photons.

This reassignment preserves the foliation by evolving 3-spheres while altering the causal roles of the two congruences. The resulting projection remains fully diffeomorphism invariant and yields the Schwarzschild–de Sitter spacetime as the unique vacuum representation compatible with the reassigned causal structure. The geometric structure underlying this reassignment is illustrated in Fig. 1.

Construction of the Schwarzschild–de Sitter Representation

To represent this causal reinterpretation, we take the radius of the expanding 3-sphere as a timelike coordinate r. Since each spatial slice is a 3-sphere orthogonal to r, the remaining spatial dimensions are spherically symmetric. A general line element consistent with these conditions takes the form

$$\mathrm{d}s^2 = -B(r,t)\,\mathrm{d}r^2 + A(r,t)\,\mathrm{d}t^2 + r^2\,\mathrm{d}\Omega^2. \tag{1}$$

Imposing the vacuum Einstein equations with a positive cosmological constant selects the Schwarzschild–de Sitter (SdS) metric [11]:

$$\mathrm{d}s^2 = -\frac{r}{\frac{\Lambda}{3}r^3 + \frac{2GM}{c^2} - r}\,\mathrm{d}r^2 + \frac{\frac{\Lambda}{3}r^3 + \frac{2GM}{c^2} - r}{r}\,\mathrm{d}t^2 + r^2\mathrm{d}\Omega^2. \tag{2}$$

The coordinate r is timelike provided

$$\frac{\Lambda G^2 M^2}{c^4} > \frac{1}{9}, \tag{3}$$

which requires

$$M > \frac{c^2}{3\sqrt{\Lambda G}} \approx 4.3 \times 10^{52}\ \text{kg}. \tag{4}$$

The total mass within the observed universe exceeds this bound, confirming that the SdS representation is a valid cosmological model.

The coordinate singularity at $r = 0$ in Eq. (2) is not a curvature singularity; it reflects only the degeneracy of the chosen coordinates under the causal reassignment. This construction exemplifies how a change of metric on a fixed manifold, subject to compatibility with the underlying foliation, produces a distinct but admissible Lorentzian representation of the same ontological evolution.

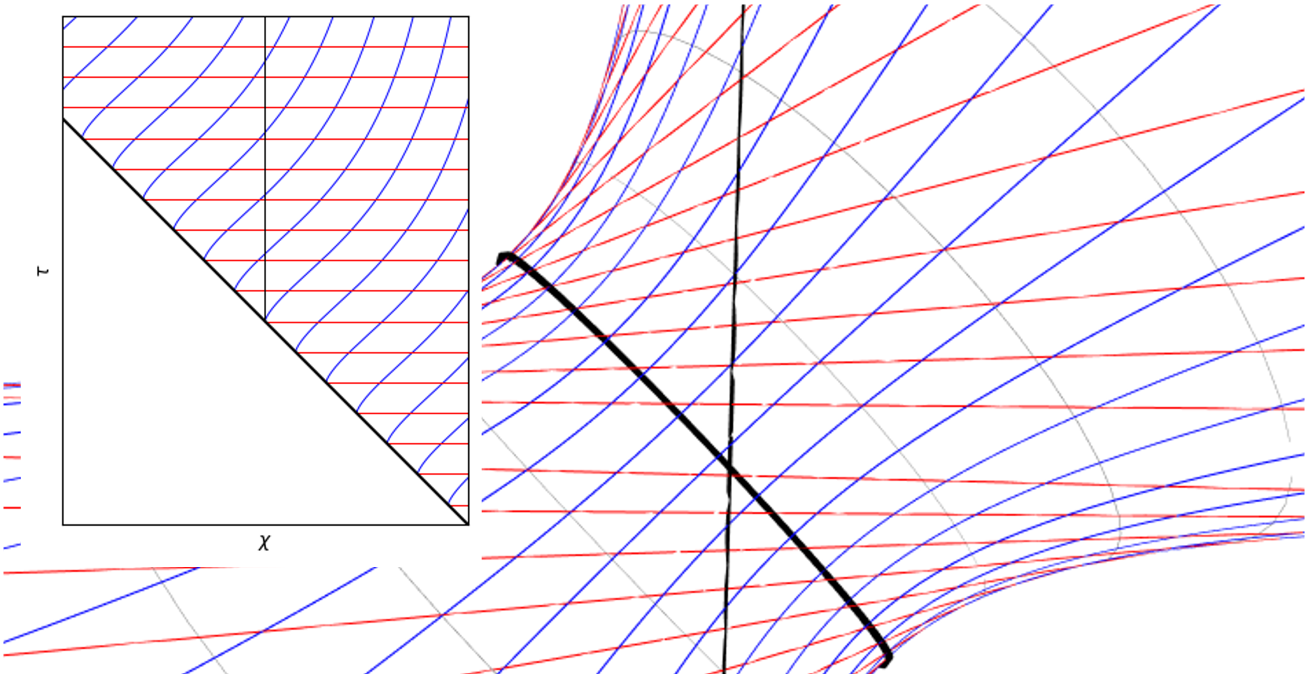

FIG. 1. Diagram illustrating the causal reassignment used to obtain the SdS cosmology from de Sitter space. The red and blue families of curves represent two distinct systems of trajectories on the de Sitter hyperboloid: the red curves correspond to the future-directed null directions associated with the causal sense of collapse-generated horizon generators, while the blue curves represent the spacelike worldlines that remain fixed on the expanding 3-spheres (constant-R curves). Under causal reassignment, the red null curves become the timelike fundamental worldlines of the SdS representation, and the blue constant-R curves become the photon (null) trajectories. The thick black curve marks the interface between regions where the curvature patterns of the two congruences differ. The inset shows the corresponding (τ, χ) coordinate grid used to express the SdS line element in the fundamental observer frame.

Fundamental Observers and the Induced Metric

Fundamental observers correspond to worldlines orthogonal to the spatial 3-spheres of constant r. In their proper frame, the line element becomes [12]

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + (\partial_\chi r)^2 \mathrm{d}\chi^2 + r^2 \mathrm{d}\Omega^2, \tag{5}$$

where

$$r(\tau, \chi) = \left(\frac{6GM}{\Lambda c^2}\right)^{1/3} \sinh^{2/3}\left(\frac{3}{2}\sqrt{\frac{\Lambda}{3}}(\tau + \chi)\right), \tag{6}$$

defined on $\tau + \chi > 0$.

The quantity $\tilde{\tau} = \tau + \chi$ defines the parameter along which the 3-sphere radius evolves. Since this parameter is tilted relative to the fundamental rest frame, the universe is *non-synchronous*: spatial slices of constant τ are Euclidean but do not coincide with cosmological spatial slices.

Exact Recovery of the Flat ΛCDM Expansion History

Equation (6) coincides exactly with the scale factor of the flat ΛCDM model:

$$\frac{a(\tilde{\tau})}{a_0} = \left(\frac{3H_0^2\Omega_{m,0}}{\Lambda c^2}\right)^{1/3} \sinh^{2/3}\left(\frac{3}{2}\sqrt{\frac{\Lambda c^2}{3}}\,\tilde{\tau}\right). \tag{7}$$

Thus the SdS model reproduces the observed redshift–distance relation and apparent expansion history of flat ΛCDM, despite being neither synchronous nor dynamically governed by matter density. The expansion arises entirely from geometric structure and causal reassignment rather than from the stress-energy content of space.

Isotropy and Observational Consistency

Although the SdS representation is anisotropic as a spacetime metric, fundamental observers comoving with the 3-spherical foliation perceive isotropy. This follows from:

1. the maximal symmetry of each spatial slice as a 3-sphere;

2. the invariance of the speed of light along null geodesics in the reassigned causal structure.

Consequently, incoming radiation is perceived isotropically, and the observational predictions coincide with those of standard FLRW cosmology. This illustrates the representational character of the SdS construction: although the Lorentzian metric differs from that of standard FLRW cosmology, the observational content perceived by fundamental observers is preserved.

Remark 14. The SdS construction demonstrates that the phenomenological success of flat ΛCDM [13–15] does not require synchronous expansion or global dynamical evolution governed by the matter density. Instead, the observed expansion history may arise as a projection of an evolving 3-sphere under a causal structure distinct from that of standard FLRW cosmology. This provides a non-synchronous, geometrically motivated cosmological model whose observational predictions match those of the standard cosmology while remaining fully consistent with the layered ontology of Cosmological Relativity.

NULL–BOUNDARY CORRESPONDENCE AND SDS ENTRY GEOMETRY

In Cosmological Relativity (CR), the ontological structure of the universe is encoded in a layered family of spatial manifolds

$$\mathcal{U} = \{\mathcal{S}_t \mid t \in \mathbb{R}\},$$

while Lorentzian spacetime metrics (M, g) arise as representational projections of this evolving geometry. The causal structure of such projections may vary, provided the layered ontology and its cosmic foliation remain fixed. This freedom permits distinct spacetime metrics—including Schwarzschild, de Sitter, and Schwarzschild–de Sitter (SdS)—to represent the same underlying geometry through different causal assignments, as shown in the SdS construction above.

The purpose of this section is to establish a structural correspondence between two geometric objects:

1. a null hypersurface arising as the limit of infalling timelike worldlines in a Lorentzian projection of CR, and
2. the initial representational slice of an SdS cosmology obtained by causal reassignment.

This correspondence is purely geometric and representational: it identifies a morphism between two projections of the same ontological layer, without asserting any physical generative relation between them.

Horizon Null Structure and One-Way Causal Ordering

Let (M, g) be a Lorentzian manifold representing a gravitational collapse scenario. From the null–temporal degeneracy theorem established in Ref. [4], the event horizon $\mathcal{H}^+$ is a null hypersurface foliated by null generators γ_λ, where:

1. each generator consists of topologically distinct, causally ordered, but metrically coincident events,
2. successive infalling timelike worldlines accumulate asymptotically onto these generators, and
3. the causal relation is one-way: earlier infalling worldlines lie in the past null boundary of later ones, but not conversely.

Topologically, the horizon has the structure

$$\mathcal{H}^+ \cong S^2 \times \mathbb{R}_{\geq 0},$$

with S^2 the cross-sectional sphere and $\mathbb{R}_{\geq 0}$ parametrizing the null ordering along generators. This structure will play a central role in the correspondence established below.

SdS Cosmology and Causal Reassignment

In the SdS construction above, we showed that by preserving the cosmic foliation while reassigning the causal roles of null and timelike congruences in de Sitter space, one obtains a Schwarzschild–de Sitter representation of the same layered ontology. The geometric legitimacy and structural meaning of this reassignment are clarified in the following subsection. In this construction:

$$\mathrm{d}s^2 = -\frac{r}{\frac{\Lambda}{3}r^3 + \frac{2GM}{c^2} - r}\,\mathrm{d}r^2 + \frac{\frac{\Lambda}{3}r^3 + \frac{2GM}{c^2} - r}{r}\,\mathrm{d}t^2 + r^2\mathrm{d}\Omega^2, \tag{8}$$

where r is a timelike coordinate aligned with the cosmic foliation, and the spatial slices are 3-spheres of radius r.

In this representation:

1. the fundamental cosmological rest frame is defined by one bundle of null generators of the de Sitter hyperboloid;
2. the complementary congruence becomes the null (photon) congruence in the SdS projection;
3. the foliation by 3-spheres is preserved;
4. the evolution of r follows the $\sinh^{2/3}$ law matching flat ΛCDM.

Metric Reassignment and Representational Freedom on a Fixed Manifold

The causal reassignment employed in the SdS construction rests on a structural distinction that is standard in differential geometry yet rarely articulated in the relativistic literature. A smooth manifold M is, by definition, a topological space equipped with a differentiable atlas; it carries no intrinsic geometric or causal structure. Lorentzian geometry arises only after the choice of a metric, that is, after specifying a smooth assignment to each tangent space T_pM of a symmetric bilinear form of signature $(-, +, +, +)$. The null cones, causal relations, and classification of tangent directions as timelike, null, or spacelike are therefore features of the metric g, not of the manifold itself.

For a fixed Lorentzian metric g, the null directions at each point are invariant under coordinate transformations: changing charts cannot alter the light cone defined by g. However, there is no requirement that a given manifold admit only one such metric. Replacing g with a distinct Lorentzian metric g' on the same manifold M alters the causal structure while leaving the underlying topology and differentiable structure unchanged. The pairs (M, g) and (M, g') thus represent the same manifold endowed with inequivalent Lorentzian geometries. This is entirely orthodox: the freedom to equip a single manifold with distinct Riemannian or Lorentzian metrics is foundational in differential geometry, and no part of the manifold structure is affected by such a replacement.

The causal reassignment carried out in this work is a highly constrained instance of this general freedom. Rather than introducing an arbitrary new metric, we consider two Lorentzian structures g and g' that share the same foliation by spacelike hypersurfaces corresponding to the layered ontology $\{\mathcal{S}_t\}$. The foliation is held fixed, while the causal classification of certain congruences threading this foliation is altered. Specifically, a congruence that is null with respect to g may become timelike with respect to g', and a congruence that is spacelike in one representation may serve as the null (photon) congruence in another, provided the resulting (M, g') retains smoothness and Lorentzian signature. In this sense, the SdS construction can be viewed as a representational change of causal roles among congruences, implemented through a metric reassignment rather than through any modification of the manifold or the ontological layering.

This has a clean group-theoretic interpretation. The structure group of the Lorentzian frame bundle is the Lorentz group $O(1, 3)$, and altering the causal roles of congruences corresponds to passing between inequivalent representations of this group on the tangent bundle. In the de Sitter case, the isometry group $SO(4, 1)$ acts transitively on the de Sitter hyperboloid and admits a discrete symmetry that exchanges its two independent families of null generators. The reassignment that underlies the SdS construction is precisely such a discrete exchange between complementary congruences. At the level of representation theory, the transition from the de Sitter metric to the SdS metric amounts to selecting a different Lorentzian structure on the same underlying manifold that remains compatible with the same foliation and the same action of the relevant symmetry groups.

This perspective also clarifies the conceptual economy of the construction. The reassignment does not invoke higher dimensions, additional fields, or modifications of Einstein's equations. The manifold M remains fixed; only the metric chosen to represent the causal structure is changed. From this viewpoint, the question addressed in the SdS construction is simply: given a manifold equipped with a fixed layered ontology, how many physically distinct Lorentzian metrics g are compatible with that ontology and its cosmic foliation? The answer is: more than one. The SdS metric arises as one such admissible choice, selected by requiring compatibility with the horizon-induced temporal orientation identified in gravitational collapse.

In this sense, a Lorentzian metric is equivalently understood as a choice of equivalence class of coordinate charts related by local Lorentz transformations, and replacing g with g' is a change of that equivalence class. The manifold itself is indifferent to which metric it carries; it is only the representational structure—through which physical observables are read—that changes. The SdS cosmology therefore exemplifies how distinct Lorentzian representations of the same underlying manifold may encode different but admissible causal assignments, while preserving the ontological content encoded in the cosmic foliation.

Null-Boundary Correspondence

We now formalize the relationship between the null horizon structure and the SdS cosmic entry slice.

Theorem 3 (Null-Boundary Correspondence in CR)**.** *Let (M, g) be a Lorentzian projection of a CR ontology in which a null hypersurface $\mathcal{H}^+$ arises as the limit of infalling timelike worldlines. Let (M', g') be an SdS representation of the same ontology obtained via causal reassignment while preserving the cosmic foliation.*

Then there exists a smooth map

$$\Psi : \mathcal{H}^+ \longrightarrow \Sigma_0 \subset M',$$

where Σ_0 is the initial 3-sphere of the SdS foliation, such that:

1. *Ψ maps each horizon generator γ_λ to a unique fundamental worldline in (M', g');*
2. *the S^2 cross-sections of $\mathcal{H}^+$ map diffeomorphically onto the S^2 cross-sections of Σ_0;*
3. *the null ordering along generators corresponds to the SdS timelike ordering along r under the reassigned causal structure;*
4. *Ψ preserves the layered ontology: the ontological layer represented at $\mathcal{H}^+$ is the same as that represented at Σ_0.*

Thus $\mathcal{H}^+$ and Σ_0 are representationally equivalent descriptions of the same ontological layer of CR under distinct causal assignments.

Proof. Because $\mathcal{H}^+$ is a null hypersurface foliated by generators with one-way causal ordering, each generator defines a unique null direction consistent with the causal cones of (M, g). Under CR causal reassignment, these null directions are reinterpreted as the timelike fundamental congruence defining the SdS rest frame.

The S^2 cross-sectional spheres of $\mathcal{H}^+$ are diffeomorphic to those of Σ_0 since both arise as projections of the same cosmic layer $\mathcal{S}_{t_0}$. The layered ontology ensures that the foliation parameter t is fixed across all projections; thus both $\mathcal{H}^+$ and Σ_0 represent the same ontological layer.

Finally, causal reassignment preserves diffeomorphism invariance while altering the metric interpretation of null and timelike directions. As discussed in the preceding subsection, this corresponds to selecting a distinct Lorentzian structure g' on the same underlying manifold M, leaving the foliation and ontological layering unchanged. Therefore, the mapping Ψ exists, is smooth, and satisfies the listed properties. □

Corollary 4. *Under the mapping of Theorem 3, the SdS timelike radius r evolves along the reassigned null direction of $\mathcal{H}^+$, recovering the $\sinh^{2/3}$ expansion law for $r(\tilde{\tau})$ as shown in Eq.* (6).

Remark 15. The correspondence established here is purely representational. It identifies two distinct Lorentzian metrics as projections of the same underlying ontological layer of CR. No physical generative claim is implied. The result demonstrates that null boundaries and SdS entry slices are diffeomorphic representations of the same ontological state under different causal assignments, consistent with the principles of Cosmological Relativity.

Remark 16 (Horizon-Selected Temporal Orientation and Closure of the CR Framework). The SdS construction developed above shows that the fundamental timelike congruence of the cosmology is precisely the causal image of the null direction selected by the event horizon in gravitational collapse. In the accompanying work on the null–temporal degeneracy of event horizons, it was shown that every valid spacelike slicing reaches the horizon only in the infinite-time limit, even though all infalling observers arrive there in finite proper time. The horizon therefore determines a unique limiting temporal orientation that is generically non-orthogonal to any spacelike hypersurface.

Within CR, this very orientation becomes the timelike direction of cosmic evolution in the SdS representation, and the metric reassignment described in the preceding subsection provides the technical mechanism by which this causal direction is promoted to the fundamental congruence of the cosmology. Thus the SdS cosmology is not merely compatible with the horizon structure proved in GR, but is in fact the natural representational outcome of it: the causal structure of the horizon motivates the non-orthogonal, non-synchronous cosmic time that CR introduces. This establishes the SdS cosmology as the natural representational outcome of the horizon-selected causal structure.

Remark 17 (No-Hair Structure at Null Boundaries). The correspondence established in Theorem 3 aligns naturally with the structural content of the classical no-hair theorem for stationary black holes [16–18]. In any Lorentzian projection of CR, a stationary exterior region is fully characterised—up to diffeomorphism—by the invariants (M, J, Q), and the event horizon appears as a null S^2 endowed with a single degenerate causal direction. All infalling timelike worldlines asymptotically accumulate along this generator structure, and no additional free data are encoded on the null boundary. Under CR's causal reassignment, these same invariants determine the fundamental congruence and large-scale parameters in the SdS cosmological projection. The no-hair property therefore provides a structural explanation for why the SdS representation depends only on these global quantities: they are precisely the diffeomorphism-invariant data encoded on the null boundary of the exterior spacetime.

Remark 18 (Empirical Distinguishability). The SdS cosmological model constructed above reproduces the late-time expansion history of flat ΛCDM exactly. However, the two frameworks diverge in the early universe. In standard ΛCDM, the expansion rate during the radiation-dominated epoch is governed by the stress-energy content of the primordial plasma, which determines the sound horizon and shapes the acoustic peak structure of the CMB anisotropy spectrum [15]. In the SdS model, the expansion is governed entirely by geometric structure and causal reassignment; radiation density plays no role in setting the expansion rate at any epoch. This structural difference implies that the acoustic scale, the sound horizon at recombination, and the detailed form of the CMB anisotropy spectrum may differ between the two frameworks. A rigorous derivation of the CMB predictions within the SdS framework requires modelling sound-horizon evolution within the 3-sphere ontology and projecting it into the observational frame. The present analysis establishes only that such a comparison is, in principle, possible and that the two projections are empirically distinguishable.

Remark 19 (Structural Parallels with Two-State Quantum Systems). The de Sitter hyperboloid admits two complementary families of null generators, and the causal reassignment underlying the SdS construction exchanges the roles of these families: one becomes the timelike congruence defining the fundamental rest frame, while the other becomes the photon congruence. This discrete exchange between complementary bases is formally analogous to the transformation between two measurement bases in a two-state quantum system. Although no physical identification is implied, the structural similarity suggests a potential geometric connection between the causal geometry of de Sitter space and abstract two-state systems, warranting further mathematical investigation.

Remark 20 (Positive Curvature and Intrinsic Lorentzian Structure). Among the maximally symmetric solutions of $R_{\mu\nu} = \Lambda g_{\mu\nu}$, only the case $\Lambda > 0$ yields a four-dimensional manifold whose Lorentzian signature and null-cone structure arise intrinsically from its curvature without analytic continuation. The de Sitter hyperboloid is therefore the unique maximally symmetric Lorentzian manifold with a real coordinate basis, whereas the cases $\Lambda = 0$ or $\Lambda < 0$ require an imposed identification of time to obtain Lorentzian signature. This geometric fact underlies the role of de Sitter space in the CR/SdS construction and clarifies why the causal reassignment procedure naturally employs positive curvature in generating the SdS representation. (See also [19] for a detailed analysis of the maximally symmetric solutions.)

SYNTHESIS AND STRUCTURAL CLOSURE OF THE TWO-PAPER FRAMEWORK

Taken together, the results of this paper and its companion on the null–temporal degeneracy of event horizons reveal a single continuous geometric structure underlying both gravitational collapse and cosmological expansion. In the first paper, working entirely within general relativity, it was shown that no admissible spacelike slicing corresponding to any finite temporal parameter can intersect an event horizon, even though every infalling timelike worldline reaches the horizon in finite proper time. The horizon therefore determines a unique limiting causal direction inherited by all exterior temporal functions. This direction is null, generically non-orthogonal to any spacelike hypersurface, and implies that no finite-time slice of the external universe can contain a curvature singularity or support a point-mass configuration. These statements are consequences solely of the Lorentzian causal structure of general relativity.

In the present work, Cosmological Relativity (CR) augments general relativity by introducing a global cosmic foliation and relaxing the requirement of hypersurface orthogonality that underlies the standard FLRW model. Within this augmented framework, the Schwarzschild–de Sitter (SdS) cosmology arises by causal reassignment of de Sitter space, implemented via the metric representational freedom detailed in the manifold representational freedom and metric reassignment subsection above. Crucially, the timelike fundamental congruence of the SdS representation is found to be precisely the representation of the null direction singled out by the event-horizon structure in the first paper, while the evolving 3-sphere radius obeys the exact $\sinh^{2/3}$ expansion law of the empirically successful flat ΛCDM model. The SdS representation is non-synchronous and non-orthogonal, yet observationally isotropic, and remains fully consistent with the Einstein field equations.

The two results therefore meet in a single geometric point: the null direction selected asymptotically at the horizon in general relativity becomes, under CR's projection principle, the cosmic temporal direction generating the SdS cosmology. In this way, the cosmological time assumed by CR is not arbitrary but is determined by the causal structure of gravitational collapse, and the SdS cosmology is the natural representational outcome of that structure. The augmentation introduced by CR thus becomes self-justifying: the causal orientation that CR assumes at the outset is precisely the one that general relativity identifies in the horizon limit, and the SdS cosmology closes the explanatory loop by exhibiting the observational expansion history of the real universe.

This unified structure has two further consequences. First, because no finite ontological layer may contain a point-mass configuration, the SdS cosmology is compatible with the requirement that density remain finite on every finite cosmic slice. Second, because the SdS expansion is observationally indistinguishable from flat ΛCDM at late times but differs predictively at early times, the framework provides new empirical avenues for distinguishing representational cosmologies while retaining full agreement with tested predictions of general relativity.

Accordingly, the two-paper sequence shows that the large-scale expansion compatible with current observations is not merely consistent with general relativity but follows from its global causal structure when that structure is allowed to motivate the non-orthogonal cosmic foliation of CR. The resulting picture strengthens, rather than modifies, general relativity: the empirically successful cosmological expansion appears not as an additional assumption about the universe but as a structural consequence of the theory's own horizon geometry. The framework developed here therefore situates the observed cosmological model within the internal logic of general relativity, yielding a representation that is both empirically testable and geometrically compelled.

OUTLOOK: STRUCTURAL UNITY ACROSS COLLAPSE AND COSMOLOGY

The two papers in this sequence reveal a unified geometric thread running through the most extreme regimes of general relativity. The null–temporal degeneracy established in the first paper shows that the event horizon imposes a universal constraint on admissible temporal functions: no finite-time slice of the external universe can intersect a horizon generator, even though every infalling worldline reaches the horizon in finite proper time. The resulting null direction provides a canonical temporal orientation determined solely by the global causal properties of general relativity.

In the present paper, that horizon-selected direction becomes the fundamental temporal structure within Cosmological Relativity, where the causal reassignment of de Sitter space produces the Schwarzschild–de Sitter cosmology. The evolving 3-sphere radius of this cosmology reproduces the observed $\sinh^{2/3}$ expansion of flat ΛCDM while permitting early-time deviations that render the model empirically testable. Crucially, the causal direction employed in the CR/SdS representation is not chosen arbitrarily, but is exactly the limiting direction identified in the first paper.

The representational freedom clarified in this work suggests a broader space of Lorentzian structures compatible with a fixed cosmic foliation, and understanding this space may illuminate further structural relationships between collapse geometry and cosmological evolution. In particular, the admissible causal reassignments encountered here point toward associated symmetry correspondences within the relevant structure groups—most notably the Lorentz group $O(1,3)$ governing local frame transformations and the de Sitter isometry group $SO(4,1)$ whose discrete symmetries exchange the complementary null rulings of the de Sitter hyperboloid. Identifying and classifying such group-theoretic correspondences would further clarify the geometric scope of the representational freedom that underlies the SdS construction.

Together, the two papers establish that the large-scale cosmological expansion compatible with current observations is motivated directly by the global causal structure of general relativity. In this sense, the SdS cosmology derived here is not a modification of Einstein's theory but a representational consequence of its horizon geometry. The framework invites several avenues for further investigation, including a detailed analysis of early-time observational signatures, the treatment of perturbations within the SdS representation, and the extension of the null-boundary correspondence to more general collapse scenarios. These directions offer the prospect of deepening our understanding of how general relativity's global structure constrains the possible representations of cosmic evolution.

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